Definition:

Interference is the constructive or destructive combination of waves. It occurs among waves of the same wavelength and polarization. The fields of interfering waves sum and then the time-averaged irradiance is observed. The time-averaged irradiance of combined waves with different wavelengths and/or polarizations is the sum of the individual iradiances. Note that interfering waves must be coherently related, i.e. the sources must have a definite phase relationship, as in a laser and have the same polarization.

Consider a z-propagating optical wave. The field is

$$E(z,t) = A\cos(-\omega t + kz + \Theta).$$

However, the irradiance (time-averaged) is proportional to $A^2/2$ and the phase information is lost. Now consider the combination of the fields for two z-propagating waves with the same polarization and wavelength. The total field is

$$E(z,t) = A_a\cos(-\omega t + kz + \Theta_a) + A_b\cos(-\omega t + kz + \Theta_b).$$

The phase difference between the waves is $(\Theta_a - \Theta_b)$ and the irradiance will depend on the amplitudes and on this phase difference as shown in the figure.

A phase difference of $(\Theta_a - \Theta_b) = 0$ radians give constructive interference. The total field is

$$E\,(z,t) = A_a \cos(-\,\omega t + kz\,) + A_b \cos(-\,\omega t + kz) = (A_a + A_b) \cos(-\,\omega t + kz).$$

A phase difference of $(\Theta_a - \Theta_b) = \pi$ radians gives destructive interference. The total field is

$$E\,(z,t) = A_a \cos(-\,\omega t + kz + \Theta_a) + A_b \cos(-\,\omega t + kz + \Theta_a - \pi).$$

$$E\,(z,t) = A_a \cos(-\,\omega t + kz + \Theta_a) - A_b \cos(-\,\omega t + kz + \Theta_a)$$

$$E\,(z,t) = (A_a - A_b) \cos(-\,\omega t + kz + \Theta_a).$$

Hence, the maximum irradiance of the summed waves will be proportional to $(A_a + A_b)^2/2$ and the minimum irradiance will be proportional to $(A_a - A_b)^2/2$. A general phase difference gives partial constructive or destructive interference and an intermediate irradiance.

The determination of interference effects is simplified by using the phasor form of the field. Again, consider the case of waves with the same propagation vector. The two-z-propagating plane waves with the same amplitude, polarization, and wavelength, but different phases. The time-average irradiances of the individual waves are proportional to $|E_1(z)|^2 = |E_2(z)|^2 = A^2$. The sum of the electric fields is

$$E\,(z,t) = E_1\,(z,t) + E_2\,(z,t)$$
$$= A \cos(-\,\omega t + kz + \Theta_a) + A \cos(-\,\omega t + kz + \Theta_b).$$

The phasor field is

$E(z) = A \exp(jkz + j\Theta_a) + A \exp(jkz + j\,\Theta_b)$

$E(z) = A \exp(jkz + j\Theta_a) \{1 + \exp[-j(\Theta_a - \Theta_b)]\}.$

If $(\Theta_a - \Theta_b) = 0$, then

$E(z) = 2A \exp(jkz + j\Theta_a)$ and $|E(z)|^2 = 4A^2$ (constructive interference)

If $(\Theta_a - \Theta_b) = \pi$ radians, then

$E(z) = 0$ and $|E(z)|^2 = 0$ (destructive interference)

If $(\Theta_a - \Theta_b) = \pi/2$ radians, then

$E(z) = A(1 - j) \exp(jkz + j\Theta_a)$ and $|E(z)|^2 = (A^2/2)$

Now, consider two waves with different propagation directions. The two plane waves with the same amplitude, polarization, and wavelength, but different propagation vectors. The sum of the electric fields is

$E\,(\mathbf{r},t) = A \cos(-\,\omega t + ky/\sqrt{2} + kz/\sqrt{2}) + A \cos(-\,\omega t - ky/\sqrt{2} + kz/\sqrt{2})$.

The phasor field is

$E(\mathbf{r}) = A \exp(+\,jky/\sqrt{2} + jkz/\sqrt{2}) + A \exp(-\,ky/\sqrt{2} + kz/\sqrt{2})$

$E(\mathbf{r}) = A \exp(jkz/\sqrt{2})[\exp(+\,jky/\sqrt{2}) + \exp(-\,jky/\sqrt{2})]$

$E(\mathbf{r}) = A \exp(jkz/\sqrt{2})[2 \cos(ky/\sqrt{2})]$.

Then, $|E(\mathbf{r})|^2 = 4A^2 \cos^2(ky/\sqrt{2})$.

Interferometers:

Optical instruments can be formed which use interference to measure phase. Interferometers typically operate by separating a wave (to establish coherence) and then recombining the components after they have propagated along different paths. Any differences in optical path length are converted into an irradiance modulation.

Consider the case of converting phase information into an irradiance modulation. a plane wave at a given location is represented as

$E_1\,(z_0,t) = A_1 \cos(-\,\omega t + \Theta_1)$. (Note that Θ_1 includes the kz_0.)

The phasor field is $E_1(z_0) = A_1 \exp(j\Theta_1)$ and the irradiance is proportional to

$|E_1|^2 = A^2$ which is not a function of Θ_1 (the phase information is lost).

Combining E_1 with a reference wave of the same polarization and wavelength gives

$E_T(z_0,t) = E_1(z_0,t) + E_2(z_0,t) = A_1 \cos(-\omega t\ \Theta_1) + A_2 \cos(-\omega t + \Theta_2)$ and

$E_T(z_0) = A_1 \exp(j\Theta_1) + A_2 \exp(j\Theta_2)$

The irradiance is proportional to

$|E_T(z_9)|^2 = E_T E_T^*$
$= [A_1 \exp(j\Theta_1) + A_2 \exp(j\ \Theta_2)]\ [A_1 \exp(-j\Theta_1) + A_2 \exp(-j\Theta_2)]$

$|E_T(z_9)|^2 = E_1 E_1^* + E_2 E_2^* + E_1 E_2^* + E_1^* E_2$

$|E_T(z_9)|^2$
$= A_1^2 + A_2^2 + A_1 A_2 \exp[j(\Theta_1 - \Theta_2)] + A_1 A_2 \exp[-j(\Theta_1 - \Theta_2)]$

This results leads to the interference equation

$|E_T(z_9)|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\Theta_1 - \Theta_2)$ (Phase information is maintained).

If cos( ) = +1, constructive interference gives

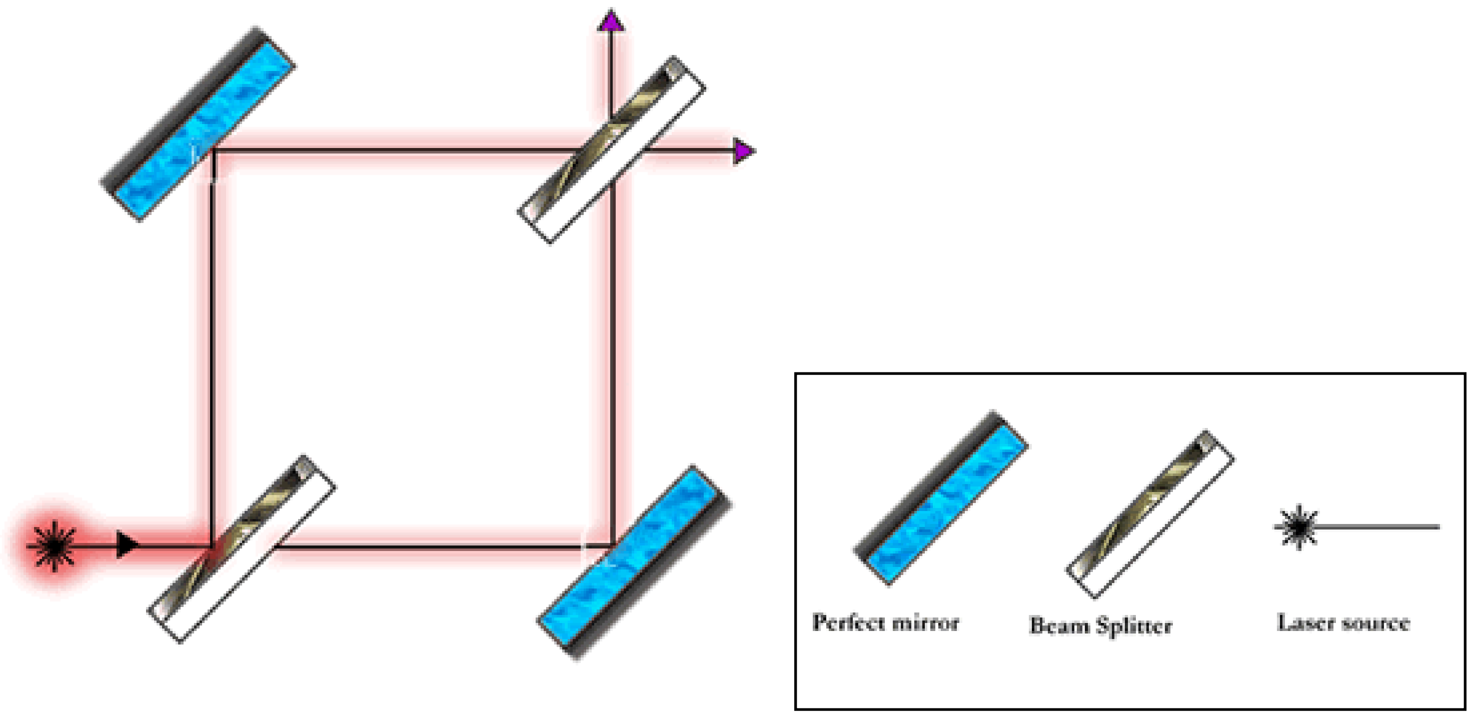
$|E_T(z_9)|^2_{MAX} = A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2$

If cos( ) = -1, destructive interference gives

$|E_T(z_9)|^2_{MIN} = A_1^2 + A_2^2 - 2A_1A_2 = (A_1 - A_2)^2$

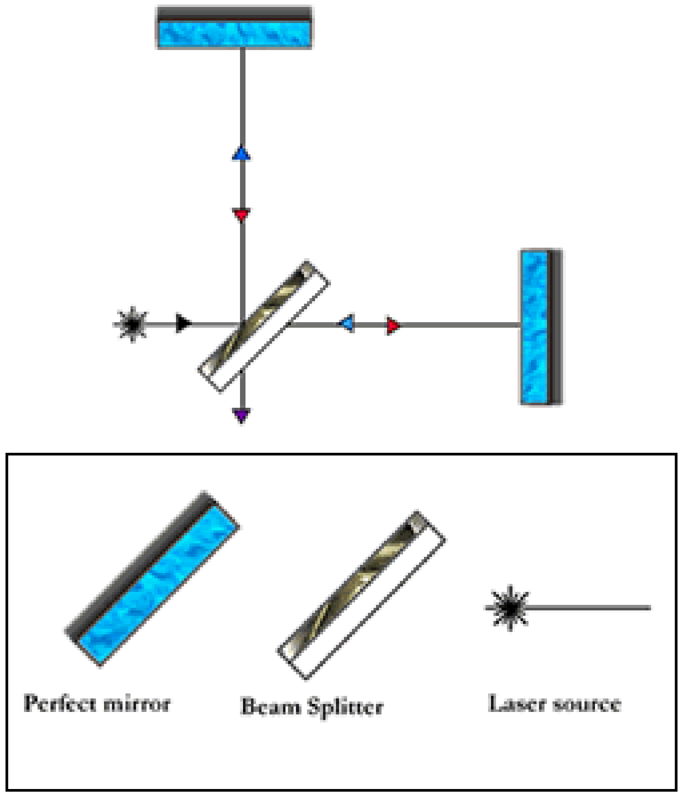
Many interferometer configurations have been developed. For interference of two optical beams, the Mach-Zehnder, the Michelson, and the Sagnac arrangements can be used. Other interferometer configurations use multiple-beam interference. The Fabry-Perot cavity is an important example.

Mach-Zehnder Two Beam Interferometer



Close Browser Window to return to Interferometry

Michelson Two Beam Interferometer



Sagnac Two Beam Interferometer